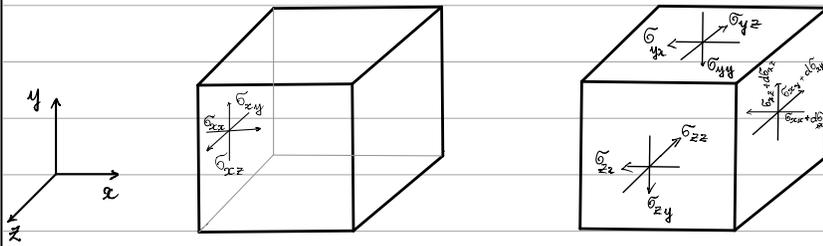


Strong form of 3D case:



σ_{ij}
i: the face that stress is acting on
j: direction of the applied stress

• Equilibrium in x direction:

$$(\sigma_{xx} + d\sigma_{xx}) dy dz - \sigma_{xx} dy dz + (\sigma_{xy} + d\sigma_{xy}) dx dz - \sigma_{xy} dx dz + (\sigma_{xz} + d\sigma_{xz}) dx dy - \sigma_{xz} dx dy + b_x dx dy dz = 0$$

external force

$$\Leftrightarrow d\sigma_{xx} dy dz + d\sigma_{xy} dx dz + d\sigma_{xz} dx dy + b_x dx dy dz = 0 \quad (1)$$

Notice that:

$$d\sigma_{xx} = \frac{\partial \sigma_{xx}}{\partial x} dx; \quad d\sigma_{xy} = \frac{\partial \sigma_{xy}}{\partial y} dy; \quad d\sigma_{xz} = \frac{\partial \sigma_{xz}}{\partial z} dz$$

(1) \Leftrightarrow

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = 0$$

Similarly, in y and z direction:

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + b_y = 0$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0$$

$$\Rightarrow \begin{cases} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = 0 \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + b_y = 0 \\ \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0 \end{cases} \quad \left(\begin{array}{l} \text{Notice that:} \\ \sigma_{xy} = \sigma_{yx} \\ \sigma_{yz} = \sigma_{zy} \\ \sigma_{xz} = \sigma_{zx} \end{array} \right)$$

\Rightarrow Static equilibrium equation in 3D:

$$\nabla_s^T \bar{\sigma} + \mathbf{b} = 0$$

in which:

$$\mathbf{b} = \begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix}; \quad \text{External force}; \quad \nabla_s =$$

body forces acting per unit volume

$$\begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ \partial/\partial y & \partial/\partial x & 0 \\ 0 & \partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & \partial/\partial z \end{bmatrix}; \quad \bar{\sigma} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix}$$



Topic: _____

Notebook

A large rectangular area with horizontal ruling lines, intended for writing notes. The lines are evenly spaced and extend across the width of the page, starting from the top header and ending at the bottom margin.