Topic: FEM 1D **Notebook**

- Objectives: to find u: at an arbitray of x

$$u''(x) = 1$$

Knowing that:

- $u(1) = 0$
- $\partial u/\partial x|_{x=0} = 0$
- $\Omega = [0, 1]$

- Residual method:

$$R(x) = \dots = 0$$

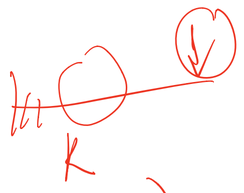
$$\Rightarrow \int_\Omega R(x) \underset{\substack{\uparrow \\ \text{arbitray}}}{v_i} dx = 0$$

$$(uv)' = u'v + uv'$$

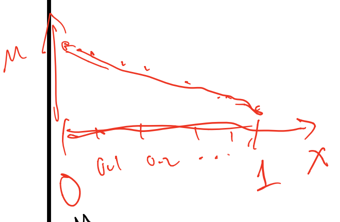
$$\int (uv)' = \int u'v + \int uv'$$

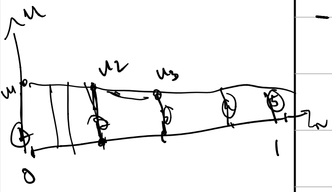
$$\Rightarrow uv = \int u'v + \int uv'$$

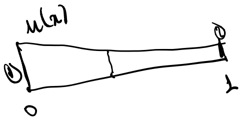
$$\Rightarrow \int u'v = -uv + \int uv'$$



$$u = g(x)$$







1) Strong form:

$$-\frac{d}{dx}\left(\cancel{c(x)}^{1} \frac{du}{dx}\right) = f(x) \quad (1)$$

Multiplying 2 sides of (1) with a test function $v(x)$ then integral

$$\int_\Omega -\frac{d}{dx}\left(c(x) \frac{du}{dx}\right) v(x)\, dx = \int_\Omega f(x) v(x) dx \quad (2)$$

$\rightarrow u$

Note: • v : virtual displacement, a bit movement from u but need to satisfy $v(0) = 0$ at fixed end

• $C \approx D \rightarrow$ stiffness matric $(K = \int_\Omega BDB\, d\Omega$

This is the **weak form**. If (2) is true for every $v(x)$ then we can get back to the strong form (1)

2 Integration by part : For "any" $v(x)$ with $v=0$ at fixed end

$$\underbrace{\int_\Omega c(x) \frac{du}{dx} \frac{dv}{dx} dx}_{\int u'v'} - \underbrace{\left[c(x) \frac{du}{dx} v(x)\right]\Big|_\Omega}_{-uv} = \int_\Omega f(x) v(x)\, dx \quad (3)$$

$v(x)$ at fixed end $= 0$) Dirichlet BC
$du/dx = 0$ at free end

3 Garlerkin method:

- Start with a continuous weak form
- Change that continuous weak form to a discrete one

$$KU = F$$

(Instead of a function unknown, I want to have n-unknowns. I will give a discrete equation, which will eventually be $KU = F$

- Choose trial functions (basis /shape function): $N_1(x), N_2(x) \dots, N_n(x)$:
- Approximate solution:

$$u(x) = u_1 N_1(x) + u_2 N_2(x) + \dots + u_n N_n(x)$$

$u_1, u_2, \dots, u_n$: N unknowns

- Choose test functions $v_1(x), v_2(x), \dots, v_n(x)$. Each $v_i(x)$ gives 1 equation. Thus, we get n equations

⇒ A square matrix, a linear system : $\mathbb{K}\,\mathbb{u} = \mathbb{F}$

NOTE:

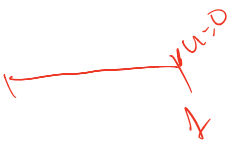
- Galerkin only applied weak form to trial & test func not to the real (continuous) weak form for a whole a lot of v

Weak form → Galerkin → Choose $\begin{cases} N_1, \dots, N_n \\ V_1, \dots, V_n \end{cases}$ ⇒ $\mathbb{K}\mathbb{U} = \mathbb{F}$

very often they are the same

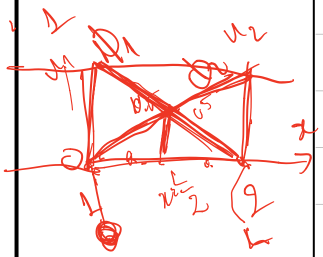
$$-\frac{d}{dx}\left(c(x)\frac{du}{dx}\right) = f(x) \Rightarrow \int c\,\frac{du}{dx}\frac{dv}{dx}\,dx = \int f(x)v(x)\,dx \quad (4)$$

STRONG WEAK



Constraint: If $u(1) = 0$ then $v(1) = 0$

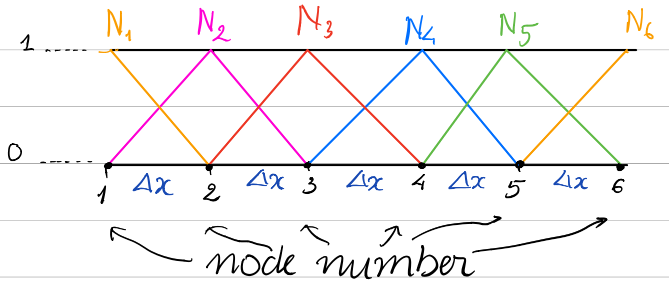
What choice will we make for ϕ_i? How do we get from all that preparation to the equation that we actually solve : $\mathbb{K}\mathbb{U} = \mathbb{F}$



$x(0) = u_1 u_1$

$\phi_1 = 1$

① Example of $\Phi(x)$ as hat function



Keep N_i simple

$u(x) = u_1\Phi_1 + u_2\Phi_2$

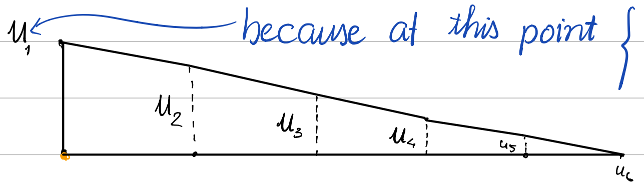
$\cdot x = 0 : \phi_1 = 1$, $\phi_2 = 0$

$\Rightarrow u(x=0) = u_1$

$\cdot x = L$

Approximation :

$$u(x) = u_1 N_1(x) + u_2 N_2(x) + \dots + u_6 N_6(x)$$



because at this point $\begin{cases} N_1 = 1 \\ N_{2\to6} = 0 \end{cases}$: shape function are "local"

. FEM : we just decide N_i, Galerkin gives us a system of equations

Where do the equations come from?

$\varepsilon = \frac{\partial u}{\partial x} = \nabla \phi \, d$

$\varepsilon = B . d$

B^T

$v = \phi$

~~$v' = \phi'$~~

$v' = \phi' = B'$

D

Weak form :

$$\int_0^1 c(x) \frac{du}{dx} \frac{dv}{dx} \, dx = \underbrace{\int_0^1 f(x) v_i(x) \, dx}_{} \qquad F = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_6 \end{bmatrix}$$

Assume : $f(x) = 1$; $c(x) = 1$; $u(1) = 0$; $\partial u / \partial x |_{x=0} = 0$

($\Rightarrow$ Equation: $u''(x) = 1 \Rightarrow u = \frac{x^2}{2} + (Cx + D)$)

Choose test functions for weak form:

$\int_0^1 N_1 \, dx$

$J[\underbrace{w_1 N(\xi_1) + w_2 N(\xi_2)}_{\text{Gauss Integration}}] \leftarrow \int_{-1}^{1} N_1(\xi) J \, d\xi$

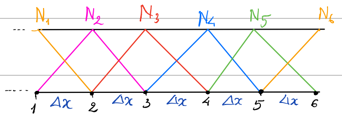
① $v = v_1(x) = N_1(x) \Rightarrow \int_0^1 (u_1 N_1' + \ldots + u_6 N_6') \frac{dN_1}{dx} \, dx = \int_0^1 N_1 \, dx$

② $v = v_2(x) = N_2(x) \Rightarrow \int_0^1 (u_1 N_1' + \ldots + u_6 N_6') \frac{dN_2}{dx} \, dx = \int_0^1 N_2 \, dx$

⋮

④ $v = v_6(x) = N_6(x) \Rightarrow \int_0^1 (u_1 N_1' + \ldots + u_6 N_6') \frac{dN_6}{dx} \, dx = \int_0^1 N_6 \, dx$

N_i & N_i' :



$$N_1 = \begin{cases} -\frac{x}{\Delta x} + 1 & ; \text{(node } 1 \to 2) \\ 0 & ; \text{others} \end{cases}$$

$$N_2 = \begin{cases} x / \Delta x & ; \text{(node } 1 \to 2) \\ -x / \Delta x + 1 & ; \text{(node } 2 \to 3) \end{cases}$$

$$N_3 = \begin{cases} x / \Delta x & ; \text{(node } 2 \to 3) \\ -x / \Delta x + 1 & ; \text{(node } 3 \to 4) \end{cases}$$

$$N_4 = \begin{cases} x / \Delta x & ; \text{(node } 3 \to 4) \\ -x / \Delta x + 1 & ; \text{(node } 4 \to 5) \end{cases}$$

$$N_5 = \begin{cases} x / \Delta x & ; \text{(node } 4 \to 5) \\ -x / \Delta x + 1 & ; \text{(node } 5 \to 6) \end{cases}$$

$$N_6 = \begin{cases} x / \Delta x & ; \text{(node } 5 \to 6) \end{cases}$$

$$N_1' = \begin{cases} -1 / \Delta x & \text{(node } 1 \to 2) \\ 0 & \text{others} \end{cases}$$

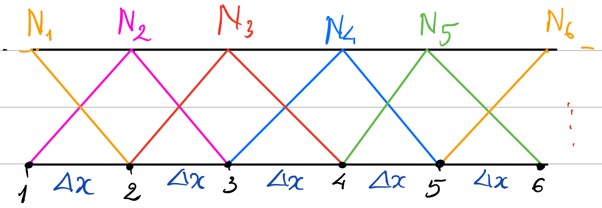
$$N_2' = \begin{cases} 1 / \Delta x & \text{(node } 1 \to 2) \\ 1 / \Delta x & \text{(node } 2 \to 3) \\ 0 & \end{cases}$$

$$N_3' = \begin{cases} 1 / \Delta x & \text{(node } 2 \to 3) \\ 1 / \Delta x & \text{(node } 3 \to 4) \\ 0 & \text{others} \end{cases}$$

$$N_4' = \begin{cases} 1 / \Delta x & \text{(node } 3 \to 4) \\ 1 / \Delta x & \text{(node } 4 \to 5) \\ 0 & \text{others} \end{cases}$$

$$N_5' = \begin{cases} 1 / \Delta x & \text{(node } 4 \to 5) \\ 1 / \Delta x & \text{(node } 5 \to 6) \\ 0 & \text{others} \end{cases}$$

$$N_6' = \begin{cases} 1 / \Delta x & \text{(node } 5 \to 6) \\ 1 / \Delta x & \\ 0 & \text{other} \end{cases}$$



- $F_1 = \int_0^1 N_1 \, dx$

= Area of the orange area

= $\Delta x/2$ => Equation 0: $\int_0^1 -(u_1 N_1' + u_2 N_2' + \ldots + u_6 N_6') \frac{dv_1}{dx} dx = \frac{\Delta x}{2}$

- $F_2 = \int_0^1 N_2 \, dx$ = Area of the purple area = Δx
- Similarly: $F_3 = F_4 = F_5 = = \Delta x$

$F_6 = \Delta x/2$

$$\frac{1}{\Delta x}\left[\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{array}\right] \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \Delta x \begin{array}{cc} & \text{node} \\ 1/2 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1/2 & 6 \end{array}$$

$$K \cdot u = F$$

in which:

$K_{11} = \int_0^1 -N_1' \frac{dV_1}{x} dx = -\int_0^{\Delta x} N_1'^2 dx = \frac{1}{\Delta x}$

$K_{12} = \int_0^1 -N_2' \frac{dv_1}{dx} dx = \int_0^{\Delta x} -N_2' N_1' dx = -\int_0^{\Delta x} \frac{1}{\Delta x}\left(-\frac{1}{\Delta x}\right) dx = -\frac{1}{\Delta x}$

$K_{13} = \int_0^1 -\phi_3' \frac{dv_1}{dx} dx = \int_0^1 -\underbrace{N_3'}\underbrace{N_1'} dx = 0$

only ≠ 0 from node 2 → 4

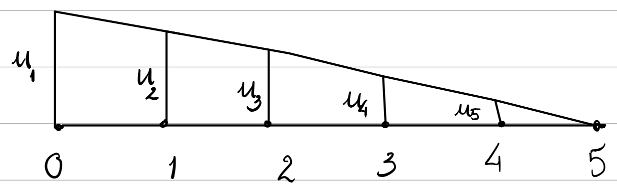
only ≠ 0 from node 1 → 2

Similarly: $K_{14} = K_{15} = K_{16} = 0$

$K_{21} = -\int N_1' N_2' dx = \int_0^{\Delta x} \frac{1}{\Delta x^2} dx = -\frac{1}{\Delta x} \Big|_0^{\Delta x} = -\frac{1}{\Delta x}$

$K_{22} = -\int_0^1 N_2^2 dx = \qquad = \frac{2}{\Delta x}$

$K_{23} = -\int_0^1 N_2' N_3' dx = K_{01} = -1/\Delta x$



$u_0 = 0.5$

$u_1 = 0.48$

$u_2 = 0.42$

$u_3 = 0.32$

$u_4 = 0.18$

$u_5 = 0$ ← automatic, not calculated

Size of K:

$$n_{node} \times n_{disp} = 6 \times 1 \Rightarrow [6\times6]$$

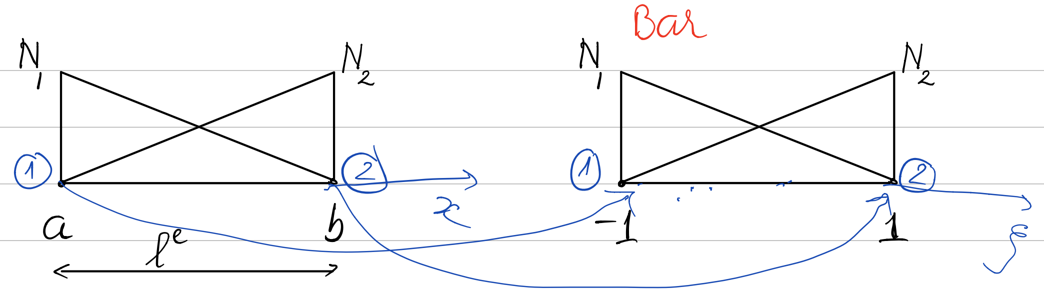
$$\begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 1 & -1 & & & & \\ 1 & -1 & 11 & -1 & & & \\ 2 & & -1 & 11 & -1 & & \\ 3 & & & -1 & 11 & -1 & \\ 4 & & & & -1 & 11 & -1 \\ 5 & & & & & -1 & 1 \end{array} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1/2 \end{bmatrix}$$

Boundary condition: $d_5 = 0$

$$\begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 1 & -1 & & & & \\ 1 & -1 & 11 & -1 & & & \\ 2 & & -1 & 11 & -1 & & \\ 3 & & & -1 & 11 & -1 & \\ 4 & & & & -1 & 11 & -1 \\ 5 & & & & & -1 & 1 \end{array} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1/2 \end{bmatrix}$$

$$\Rightarrow \begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 2 & -1 & 0 & 0 \\ 2 & 0 & -1 & 2 & -1 & 0 \\ 3 & 0 & 0 & -1 & 2 & -1 \\ 4 & 0 & 0 & 0 & -1 & 1 \end{array} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Bar



$$\begin{cases} N_1 = \dfrac{\ell^e - x}{\ell^e} \\ N_2 = \dfrac{x}{\ell^e} \end{cases}$$

$u = N_1 d_1 + N_2 d_2$ ✓

$= \dfrac{\ell^e - x}{\ell e} d_1 + \dfrac{x}{\ell e} d_2$

$\varepsilon = \dfrac{du}{dx} = \dfrac{d_2 - d_1}{\ell e}$

$\varepsilon = \underbrace{\dfrac{1}{\ell e} \begin{bmatrix} -1 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} d_2 \\ d_1 \end{bmatrix}}_{d}$

$K = \int_a^b B$

$N = [N_1 N_2]$

$N = \begin{bmatrix} \frac{1-\xi}{2} & \frac{1+\xi}{2} \end{bmatrix}$

$\underbrace{\nabla N}_{\uparrow B} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

- $N_1 = \dfrac{1-\xi}{2}$; $N_2 = \dfrac{1+\xi}{2}$
- $x = a \to \xi = -1$

  $x = b \to \xi = 1$

$\Rightarrow \xi = -1 + \dfrac{2(x-b)}{a-b}$

$\Rightarrow d\xi = \dfrac{2}{a-b} dx = \dfrac{2}{\ell^e} dx$

- $u = N_1 d_1 + N_2 d_2$ ✓ $u = N_{(\xi)} d$

  $= \dfrac{1-\xi}{2} d_1 + \dfrac{1+\xi}{2} d_2$

$\varepsilon = \dfrac{\partial u}{\partial x} =$

$\Rightarrow \dfrac{\partial u}{\partial \xi} = \dfrac{d_2 - d_1}{2}$ ✓

- $\varepsilon = \dfrac{du}{dx} = \dfrac{\partial u}{\partial \xi} \dfrac{\partial \xi}{\partial x} = \dfrac{d_2 - d_1}{2} \dfrac{2}{\ell^e}$

$\varepsilon = \dfrac{1}{\ell^e} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$

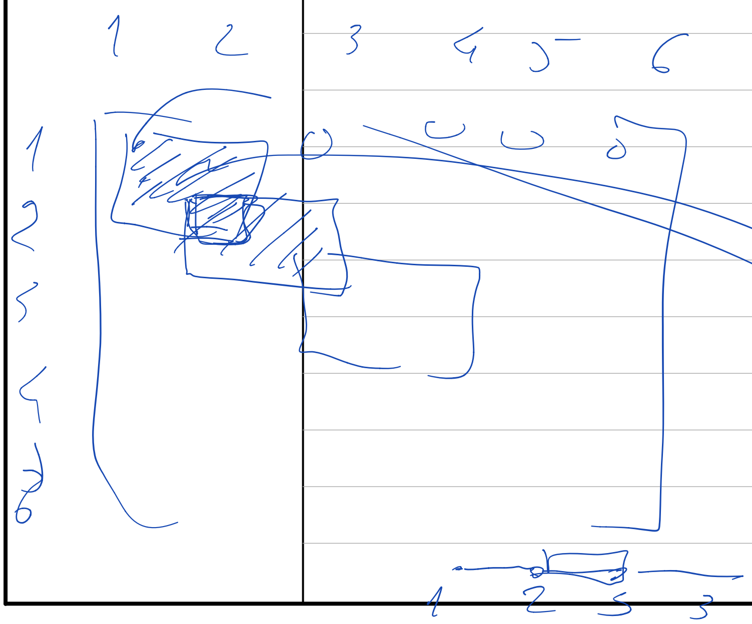
$\varepsilon = \underbrace{B}_{\to \text{constant}} d$

$K = \int_a^b \underbrace{B^{(x)}_{i} B^{(x)}_{i+1}}_{f(x)} dx$

$= J \int_{-1}^{1} \underbrace{B(\xi)\ (B(\xi))}_{f(\xi)} d\xi$

$K = J \left(w_1 f(\xi_1) + w_2 f(\xi_2) \right) = [\quad]$ 2×2

$K = \begin{array}{c|cc} & 1 & 2 \\ 1 & K_{11} & K_{12} \\ 2 & K_{21} & K_{21} \end{array}$



u -

1) Strong form (governing equations

$$-\frac{d}{dx}\left(c(x)\frac{du}{dx}\right) = f(x) \qquad (1)$$

Constraints (boundary condition) : $\begin{cases} u(1) = 0 \\ \partial u/\partial x|_{x=0} = 0 \end{cases}$

→ Objective: Find u at an arbitrary position of $x \in \Omega \equiv [0,1]$

* Remind: Strong form of small strain deformation problem?

2) Weak form.

Residual, $R = -\dfrac{d}{dx}\left(c(x)\dfrac{du}{dx}\right) - f(x) = 0$

Weighted residual method :

Multiplying 2 sides of (1) with a test function $v(x)$ then integral

$$0 = \int_{\Omega} R\, \underbrace{v_i}\, dx$$

must be satisfied everywhere

testing | function ∈ arbitrary
weighting

$$\int_{\Omega} -\frac{d}{dx}\left(c(x)\frac{du}{dx}\right)v(x)\, dx - \int_{\Omega} f(x)\, v(x)\, dx = 0$$

Note: . v : virtual displacement, a bit movement from u
but need to satisfy $v(0) = 0$ at fixed end

. $C \simeq D$ → stiffness matric $(K = \int_{\Omega} BDB\, d\Omega$

This is the **weak form**. If (2) is true for every $v(x)$ then we can get back to the strong form (1)

Integration by part : For "any" $v(x)$ with $v=0$ at fixed end

$$\int_{\Omega} c(x)\frac{du}{dx}\frac{dv}{dx}\, dx - \underbrace{\left[c(x)\frac{du}{dx}v(x)\right]\Bigg|_{\Omega}}_{-uv\ ?} = \int_{\Omega} f(x)\, v(x)\, dx \qquad (3)$$

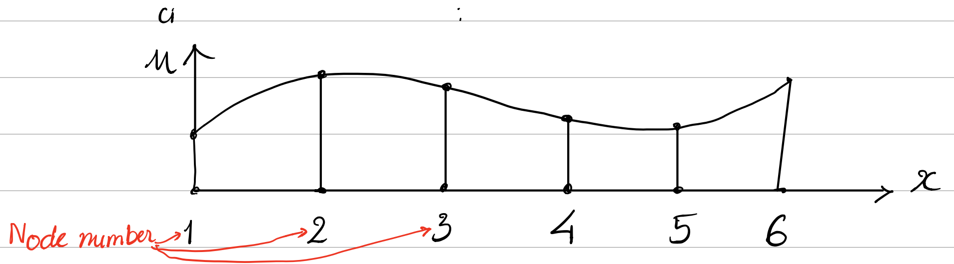
$\int u v'$

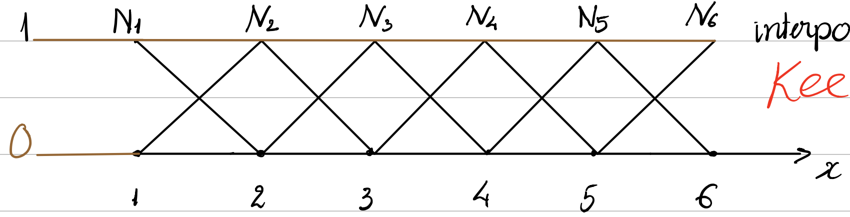
Apply boundary condition: $\begin{cases} v(x) = 0 \text{ at fixed end} \\ du/dx = 0 \text{ at free end} \end{cases}$

(3) Become :

$$\int_{\Omega} c(x)\,\frac{du}{dx}\,\frac{dv}{dx}\,dx = \int_{\Omega} f(x)\,v(x)\,dx \qquad (4)$$

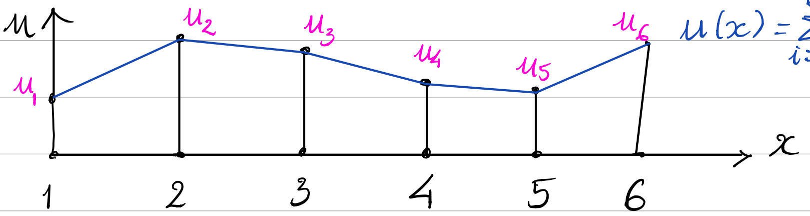
3 Spacial discretization



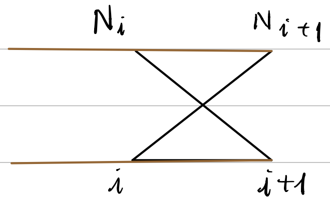


interpolation function

Keep N_i simple



$$u(x) = \sum_{i=1}^{6} N_i(x)\,d_i$$



$$N_i = \begin{cases} 0 & ; \; x \leq x_{i-1} \\ \dfrac{x - x_{i-1}}{x_i - x_{i-1}} & ; \; x_{i-1} < x < x_i \\ \dfrac{x_{i+1} - x}{x_i - x_{i-1}} & ; \; x_i < x < x_{i+1} \\ 0 & ; \; x \geqslant x_i \end{cases}$$

$$\sum_{i=1}^{6} N_i = 1$$

Approximation:

$$u(x) = N_1 d_1 + N_2 d_2 + \dots + N_6 d_6$$

4 Galerkin

- Choose test functions $v_1(x)$, $v_2(x)$, ..., $v_n(x)$. Each $v_i(x)$ gives 1 equation . Thus, we get n equations

⇒ A square matrix, a linear system : $\mathbb{K}u = \mathbb{F}$

NOTE:

- Galerkin only applied weak form to trial & test func not to the real (continuous) weak form for a whole a lot of v

$$\boxed{\text{Weak form} \rightarrow \text{Galerkin} \rightarrow \text{Choose} \begin{cases} N_1, \dots, N_n \\ V_1, \dots, V_n \end{cases} \quad \text{very often they are the same}} \Rightarrow \mathbb{K}\mathbb{U} = \mathbb{F}$$

$$-\frac{d}{dx}\left(c(x)\frac{du}{dx}\right) = f(x) \Rightarrow \int c\,\frac{du}{dx}\frac{dv}{dx}\,dx = \int f(x)v(x)\,dx \quad (4)$$

STRONG WEAK

Constraint: If $u(1) = 0$ then $v(1) = 0$

What choice will we make for ϕ_i? How do we get from all that preparation to the equation that we actually solve : $\mathbb{K}\mathbb{U} = \mathbb{F}$

Weak form :

$$\int_0^1 c(x)\frac{du}{dx}\frac{dv}{dx}\,dx = \underbrace{\int_0^1 f(x)v_i(x)\,dx} \qquad F = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_6 \end{bmatrix}$$

Assume : $f(x) = 1$; $c(x) = 1$; $u(1) = 0$; $\partial u/\partial x\big|_{x=0} = 0$

($\Rightarrow$ Equation: $u''(x) = 1 \Rightarrow u = \frac{x^2}{2} + Cx + D$)

Choose test functions for weak form:

① $v = v_1(x) = N_1(x) \Rightarrow \int_0^1 (u_1N_1' + \dots + u_6N_6')\frac{dN_1}{dx}\,dx = \int_0^1 N_1\,dx$

② $v = v_2(x) = N_2(x) \Rightarrow \int_0^1 (u_1N_1' + \dots + u_6N_6')\frac{dN_2}{dx}\,dx = \int_0^1 N_2\,dx$

⋮

④ $v = v_6(x) = N_6(x) \Rightarrow \int_0^1 (u_1N_1' + \dots + u_6N_6')\frac{dN_6}{dx}\,dx = \int_0^1 N_6\,dx$

$B_i = N_i'$

$$\int_0^1 \underbrace{\underbrace{\begin{bmatrix} N_1' \\ N_2' \\ \vdots \\ N_6' \end{bmatrix}}_{B'} \underbrace{[N_1' \; N_2' \dots N_6']}_{\mathbb{B}} \, dx}_{\mathbb{K}} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_6 \end{bmatrix}}_{u} = \underbrace{\int_0^1 \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_6 \end{bmatrix} dx}_{\mathbb{F}}$$

$$\underbrace{\int_0^1 \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_6 \end{bmatrix} [B_1 \; B_2 \dots B_6] \, dx}_{\mathbb{K}} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_6 \end{bmatrix}}_{u} = \underbrace{\int_0 ^1 \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_6 \end{bmatrix} dx}_{\mathbb{F}}$$

or

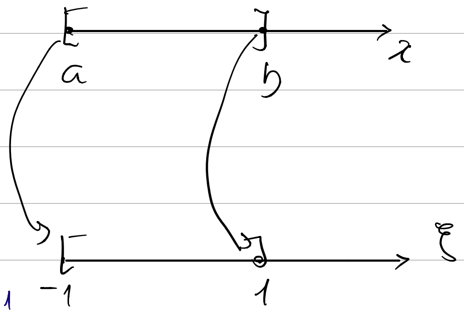
$$\mathbb{K} = \begin{bmatrix} \int_\Omega B_1 B_1 \, dx & \int_\Omega B_1 B_2 \, dx & \dots & \int_\Omega B_1 B_6 \, dx \\ \int_\Omega B_2 B_1 \, dx & \int_\Omega B_2 B_2 \, dx & \dots & \int_\Omega B_2 B_6 \, dx \\ \vdots & & & \\ \int_\Omega B_6 B_1 \, dx & \int_\Omega B_6 B_2 \, dx & \dots & \int_\Omega B_6 B_6 \, dx \end{bmatrix}$$

$$\mathbb{K} = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{16} \\ K_{21} & K_{22} & \dots & K_{26} \\ \vdots & & & \\ K_{61} & K_{62} & \dots & K_{66} \end{bmatrix}$$

- $K_{IJ} = \int_{\Omega : x = [a, b]} B_I(x) \; B_J(x) \; dx$

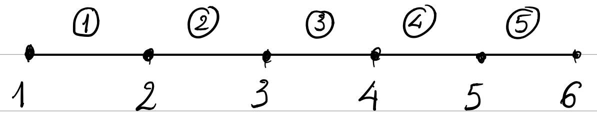
Gauss integration →

$$= \overset{\text{Jacobian}}{J} \int_{-1}^{1} \underbrace{B_I(\xi) \; B_J(\xi)}_{f(\xi)} \, d\xi$$

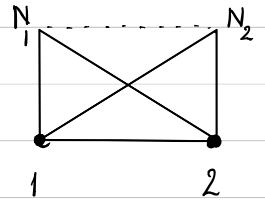


$$K_{IJ} = J \int_{-1}^{1} f(\xi) \, d\xi = J \left[\sum_{i=1}^{n_G} W_i f(\xi_i) \right]$$

⇒ Gauss Point

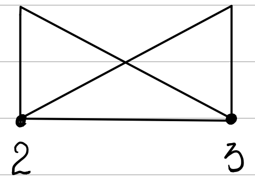


Element 1:



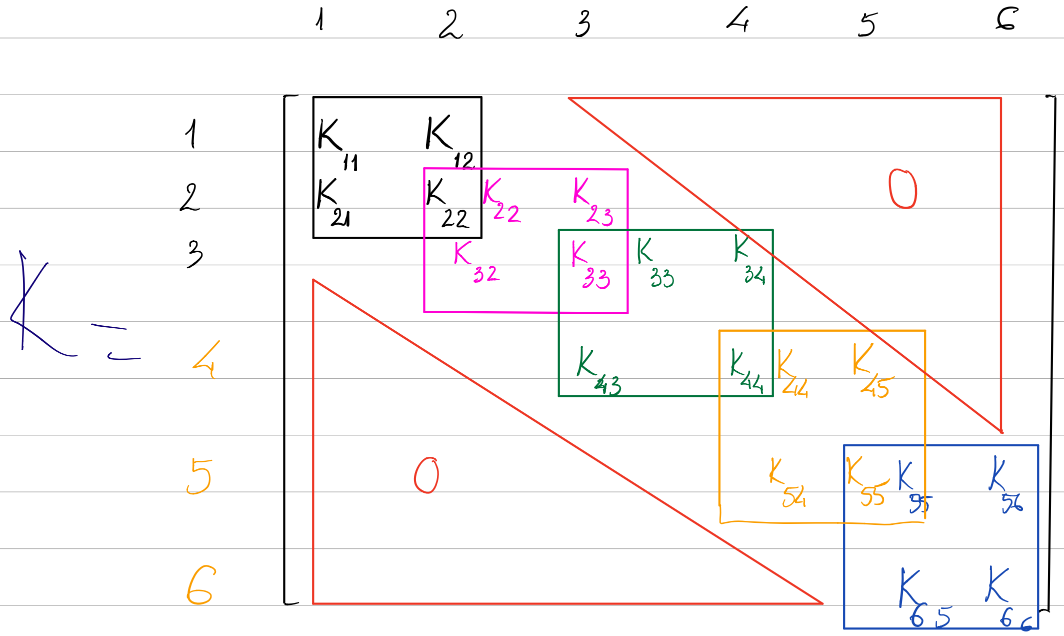
$$\begin{array}{cc} & \begin{matrix}1 & 2\end{matrix} \\ \begin{matrix}1\\2\end{matrix} & \begin{bmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{bmatrix}\end{array}$$

Element 2:



$$\begin{bmatrix}K_{22} & K_{23}\\ K_{32} & K_{33}\end{bmatrix}$$

Assemble



- We need to know which positions that stiffness of element i is contributing to the global stiffness matrix
- Besides the positions of $K_{i\,local}$ that contributed, the other position: 0

→ sparse matrix

$$\underbrace{K\;\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & & & & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}}_{K} \underbrace{\begin{bmatrix} \\ \\ \\ \\ \\ u_6 \end{bmatrix}}_{u} = \underbrace{\begin{bmatrix} \\ \\ \\ \\ \\ 0 \end{bmatrix}}_{F}$$

⇒ $u_6 = 0$

$$\boxed{\frac{\partial u}{\partial x} = 0}$$