VOLUMETRIC LOCKING

- A material is called "incompressible" when it deforms in such a fashion that $\varepsilon_v = 0$
  $\Rightarrow$ Strain tensor is purely deviatoric!
- In reality, there are no materials that are completely incompressible, but there are materials that are practically incompressible; that is, they satisfy $\varepsilon_v \to 0$.
- This is the case for materials in which the deviatoric material stiffness $\mu = G$ is much lower than the volumetric material stiffness K_v
- One example is water & fluids under "slow loading" conditions, for which the shear stiffness is negligible compared to the volumetric material stiffness, and thus the material can be treated as incompressible.
- Other examples are metals (such as steel) used in construction, when deformed in the inelastic regime, undergo straining in an almost equivolumetric fashion.
- To describe these materials, we seperate the strain tensor into 2 parts:

$$\varepsilon = \varepsilon^e + \varepsilon^p$$

- Acknowledging that $\varepsilon_v^p = 0 \Rightarrow \varepsilon^p = \varepsilon_d^p$
- Due to the fact that $\varepsilon^p \gg \varepsilon^e$ ($\to$ 10x)
  $\Rightarrow$ Response of elastoplastic metals will be near-incompressible.
- Based on the above, incompressibily (or rather near-incompressiblity) is an important aspect of FEA
- We can also establish incompressibility or near-incompressibility for a linearly elastic isotropic material by setting $\nu \to 1/2$. $\Rightarrow K_v \to \infty$

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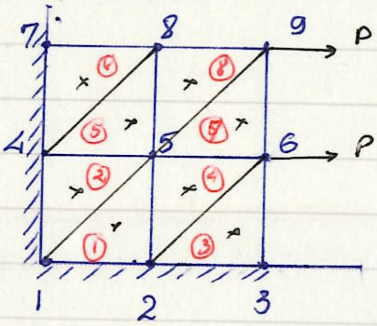
How FEM should be modified to account for incompressibility

- It turns out that incompressibity introduces constraint equations, involving the nodal displacement of the mesh.

Examples



• nodal points

x quadrature point

Both displacement restrained for nodes: 1, 2, 3, 4, 7

The mesh includes a total of 8 constant-strain triangular (CST) elements. Each elements has a single quadrature point. The nodal displacements of nodes 1, 2, 3, 4, 7 are restrained → 0.

- Vector of unrestrained DOF (u_f) would have a total of 8 components (2 displacements for each of the 4 unrestrained nodal points)
- Now, the requirement for incompressibility leads to equations demanding that we have zero volumetric strain for each location where we calculate the strains. These points are simply the quadrature points of the elements.
- For ex: element (1) (2), (3) (4) (7)

$$\varepsilon_v^{(1)} = 0 \iff [b_{vol}^{(1)}]\{u^{(1)}\} = 0 \quad ; i = 1, 2, 3, 4, 7$$

where $[b_{vol}^{(e)}]$ is a 6×1 row vector, giving the ε_v of the quadrature point of each element e, in terms of the nodal displacements of the same element.

- Now, a very interesting situation arises for this ex: due to: $\begin{cases} 8 \text{ constraint equations} \\ 8 \text{ unrestrained nodal displacement} \end{cases}$

?

⇒ The only way to satisfy this all eight constraint equations is to have $\{u_f\} = 0$

That is even though we have applied forces on our

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mesh, we will have zero displacement everywhere. due to the fact that there is a excessive number of incompressibility constraint equation! This is an extreme case of the situation where the additional constraints due to incompressibility may lead to overly stiff solution (the solution where the displacements are ~~seve~~ severely underestimated or they are even equal to zero everywhere). This situation is term « volumetric locking »

Constraint counting method

- One quick, practical way to determine whether an element will have a propensity to lock is based on constraint ratio R. That is, the number of nodal equilibrium equation (= n_o of unrestrained nodal displacements) divided by the number of ~~unrestrained nodal displacement~~ incompressibility constraints.
- If we have a finite element mesh, we need to count the number of unrestrained nodal displacements. N_f and the number of incompressibility constraints N_c. The later is simply equal to the total number of Gauss (quadrature) points in the elements of the mesh.

$$R = \frac{N_f}{N_c}$$

For plane strain analysic.

- If $R=2$: we have optimal condition (no locking)
- If $1<R\leq 2$: "overconstrained mesh" (more constraints than those corresponding to optimal behavior, and we will have a overstiffness respond
- If $R\leq 1$: we have a locking mesh (much more constraints than those corresponding to optimal behavior). These excessive constraint lead to severe locking, which may even give a solution with zero displacements everywhere, despite the fact that we apply loads on the
- $R>2$: Unlikely to occure in practice (underconstraint mesh) incompressity constraint will not be satisfied ~~any~~ where exactly