TENSOR NOTATION

1 Quantity

+ Scalar: one value → size : a, b
  eg: age, height, money
+ Vector : size & direction a_i, a_j
  eg: force
+ Tensor : linear transformation of a vector A, A_{ij}, $\mathbb{A}_{ij}$

2 Meaning of "tensor":

_ If we multiply tensor to a vector, we get another vector

Eg: $\begin{pmatrix} 3 & 1 \\ 5 & 9 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 23 \end{pmatrix}$ → $3.1 + 1.2$, $5.1 + 9.2$

_ Meaning of "linear"

Eg: $A\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1^2 + 3.2^3 \\ \sqrt{1} + 0{,}5.2^2 \end{pmatrix}$: not a linear transformation

Maybe there is non-linear transformation technique

3 Tensor Notation:

_ Vector : a_i, b_j $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$

_ Tensor : A_{ij}, B_{ij} $A = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$

Free index = 1, 2, 3 axis

_ $a_i b_i = \sum_{i=1}^{3} a_i b_i = a.b$ (inner product)

$A_{jj} = \sum_{j=1}^{3} A_{jj} = A_{11} + A_{22} + A_{33}$

_ Same index in 1 calculation → Dummy index
_ The letter jj is not important : $A_{jj} = A_{pp} = A_{mm}$

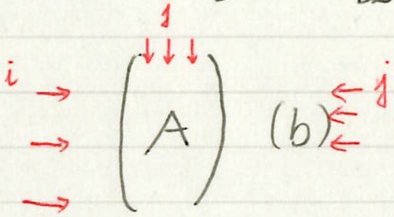
TENSOR NOTATION

4 The transformation of vector b by tensor A

$$A_{ij}\, b_j = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \sum_{j=1}^{3} A_{ij}\, b_j = A_{i1} b_1 + A_{i2} b_2 + A_{i3} b_3$$

1,2,3

$$= \begin{bmatrix} A_{11} b_1 + A_{12} b_2 + A_{13} b_3 \\ A_{21} b_1 + A_{22} b_2 + A_{23} b_3 \\ A_{31} b_1 + A_{32} b_2 + A_{33} b_3 \end{bmatrix}$$



5 Multiplying 2 tensors:

i=1, j=1   i=1, j=2

$$\begin{pmatrix} A_{11} & A_{12} + A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{pmatrix} = \begin{pmatrix} \circ & \circ & \\ & & \\ & & \end{pmatrix}$$

$$A_{ik} B_{jk} = A_{i1} B_{1j} + A_{i2} B_{2j} + A_{i3} B_{3j}$$

$$A_{ii} = \mathrm{tr}A = A_{11} + A_{22} + A_{33}$$

$$a_i b_i = \mathbf{a} \cdot \mathbf{b}$$