Healing & Decay - Rock Structure

Yield function:

$$f = \frac{\lambda - \kappa}{v_0}\left[\ln\frac{p}{p_0} + \ln\left\{1+\left(\frac{\eta}{M}\right)^2\right\}\right] - \frac{\psi-\psi_0}{v_0} + \frac{\varrho-\varrho_0}{v_0} - \varepsilon_v^p$$

Consistency condition:

$$df = 0$$

$$\Leftrightarrow \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \psi} d\psi + \frac{\partial f}{\partial \varrho} d\varrho + \frac{\partial f}{\partial \varepsilon_v^p} d\varepsilon_v^p = 0 \quad (1)$$

Evolution law:

$$\frac{d\varrho}{1+e_0} = -G(\varrho)\,\|d\varepsilon_{ij}^p\|, \quad G = a\varrho|\varrho|$$

$$\frac{d\psi}{1+e_0} = -L(\psi)\,\|d\varepsilon_{ij}^p\| + \underbrace{Q(t)\,dt}_{\text{Healing}}, \quad L = b\psi|\psi|$$

(Decaying)

Time-dependent healing:

$$\psi = \psi_{max}\left\{1 - e^{-t/t_{rej}}\right\}$$

$$\Rightarrow \frac{d\psi}{dt} = \frac{1}{t_{rej}}\psi_{max}\, e^{-t/t_{rej}}$$

$$\Rightarrow \frac{1}{1+e_0}\frac{d\psi}{dt} = \underbrace{\frac{1}{t_{rej}}\psi_{max}\left(1 - \frac{\psi}{\psi_{max}}\right)\frac{1}{1+e_0}}_{Q(t)}$$

$$\Rightarrow \frac{1}{1+e_0} d\psi = Q(t)\,dt$$

$$(1) \Leftrightarrow \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \left(L\|d\varepsilon_{ij}^p\| - Q\,dt\right) - G\|d\varepsilon_{ij}^p\| - \Lambda\frac{\partial f}{\partial \sigma_{ii}} = 0$$

$$\Leftrightarrow \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + L\Lambda\frac{\partial f}{\partial \sigma_{ij}} - Q\,dt - G\Lambda\frac{\partial f}{\partial \sigma_{ij}} - \Lambda\frac{\partial f}{\partial \sigma_{ii}} = 0$$

$$\Leftrightarrow \Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} - Q\,dt}{\frac{\partial f}{\partial \sigma_{ii}} + (G-L)\left\|\frac{\partial f}{\partial \sigma_{ij}}\right\|} \quad (2)$$

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Notice that:

$$\cdot\ d\sigma_{ij} = D^e_{ijkl} d\varepsilon^e_{kl} = D^e_{ijkl}(d\varepsilon_{kl} - d\varepsilon^p_{kl}) = D^e_{ijkl} d\varepsilon_{kl} - D^e_{ijkl} \Lambda \frac{\partial f}{\partial \sigma_{kl}} \quad (3)$$

$$(2) \Rightarrow \Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} \left(D^e_{ijkl} d\varepsilon_{kl} - D^e_{ijkl} \Lambda \frac{\partial f}{\partial \sigma_{kl}} \right) - Q\,dt}{\frac{\partial f}{\partial \sigma_{ii}} + (G-L) \left\| \frac{\partial f}{\partial \sigma_{ij}} \right\|}$$

$$\Rightarrow \Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} D^e_{ijkl} d\varepsilon_{kl} - Q\,dt}{\underbrace{\frac{\partial f}{\partial \sigma_{ii}} + (G-L) \left\| \frac{\partial f}{\partial \sigma_{ij}} \right\|}_{A} + \frac{\partial f}{\partial \sigma_{ij}} D^e_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}}$$

$$(3) \Rightarrow d\sigma_{ij} = D^e_{ijkl} d\varepsilon_{kl} - D^e_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} \frac{\frac{\partial f}{\partial \sigma_{mn}} D^e_{mnop} d\varepsilon_{op} - Q\,dt}{A + \frac{\partial f}{\partial \sigma_{ij}} D^e_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}}$$

$$= \underbrace{\left[D^e_{ijop} - \frac{D^e_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{mn}} D^e_{mnop}}{A + \frac{\partial f}{\partial \sigma_{ij}} D^e_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}} \right]}_{D^{ep}_{mnop}} d\varepsilon_{op} + \underbrace{D^e_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} \frac{Q}{A + \frac{\partial f}{\partial \sigma_{ij}} D^e_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}}}_{D^{ep}_{mnop} \cdot \frac{1}{A}} dt$$

Finally:

$$d\sigma_{ij} = D^{ep}_{mnkl} d\varepsilon_{kl} + D^{ep}_{mnkl} \frac{\partial f}{\partial \sigma_{kl}} \frac{Q}{A} dt$$

in which:

$$D^{ep}_{mnop} = \frac{D^e_{ijkl} A}{A + \frac{\partial f}{\partial \sigma_{ij}} D^e_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}}$$

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$$d\sigma_{ij} = D^{ep}_{ijkl}\, d\varepsilon_{kl} + \frac{D^{e}_{ijkl}\dfrac{\partial f}{\partial \sigma_{kl}}\dfrac{\partial Q}{\partial \sigma_{kl}}}{A}\, dt$$

$$\begin{bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ d\sigma_{12} \\ d\sigma_{13} \\ d\sigma_{23} \end{bmatrix} = \underbrace{\begin{bmatrix} D_{1111} & D_{1122} & D_{1133} & D_{1112} & D_{1113} & D_{1123} \\ D_{2211} & D_{2222} & D_{2233} & D_{2212} & D_{2213} & D_{2223} \\ D_{3311} & D_{3322} & D_{3333} & D_{3312} & D_{3313} & D_{3323} \\ D_{1211} & D_{1222} & D_{1233} & D_{1212} & D_{1213} & D_{1223} \\ D_{1311} & D_{1322} & D_{1333} & D_{1312} & D_{1313} & D_{1323} \\ D_{2311} & D_{2322} & D_{2333} & D_{2312} & D_{2313} & D_{2323} \end{bmatrix}}_{D^{ep}} \begin{bmatrix} d\varepsilon_{11} \\ d\varepsilon_{22} \\ d\varepsilon_{33} \\ d\varepsilon_{12} \\ d\varepsilon_{13} \\ d\varepsilon_{23} \end{bmatrix} + \underbrace{\begin{bmatrix} D^{t}_{11} \\ D^{t}_{22} \\ D^{t}_{33} \\ D^{t}_{12} \\ D^{t}_{13} \\ D^{t}_{14} \end{bmatrix}}_{D^{t}} dt$$

$$\underset{(6\times 1)}{\begin{bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ d\sigma_{12} \\ d\sigma_{13} \\ d\sigma_{23} \\ dt \end{bmatrix}} \Leftrightarrow \underbrace{\begin{bmatrix} D_{1111} & D_{1122} & D_{1133} & D_{1112} & D_{1113} & D_{1123} & D^{t}_{11} \\ D_{2211} & D_{2222} & D_{2233} & D_{2212} & D_{2213} & D_{2223} & D^{t}_{22} \\ D_{3311} & D_{3322} & D_{3333} & D_{3312} & D_{3313} & D_{3323} & D^{t}_{33} \\ D_{1211} & D_{1222} & D_{1233} & D_{1212} & D_{1213} & D_{1223} & D^{t}_{12} \\ D_{1311} & D_{1322} & D_{1333} & D_{1312} & D_{1313} & D_{1323} & D^{t}_{13} \\ D_{2311} & D_{2322} & D_{2333} & D_{2312} & D_{2313} & D_{2323} & D^{t}_{23} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{D^{ept}} \begin{bmatrix} d\varepsilon_{11} \\ d\varepsilon_{22} \\ d\varepsilon_{33} \\ d\varepsilon_{12} \\ d\varepsilon_{13} \\ d\varepsilon_{23} \\ dt \end{bmatrix}$$

$(6\times 6)\cdot(6\times 1) + (6\times 1)$

Triaxial test

$$\begin{bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ dt \end{bmatrix} = \begin{bmatrix} D_{1111} & D_{1122} & D_{1133} & D^{t}_{11} \\ D_{2211} & D_{2222} & D_{2233} & D^{t}_{22} \\ D_{3311} & D_{3322} & D_{3333} & D^{t}_{33} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\varepsilon_{11} \\ d\varepsilon_{22} \\ d\varepsilon_{33} \\ dt \end{bmatrix}$$