.Principles; base on the definition of Poisson ratio

$\nu = \frac{-\varepsilon_h}{\varepsilon_v}$

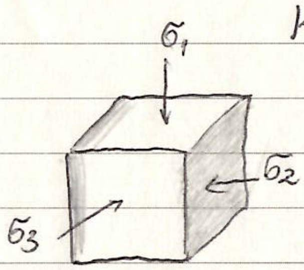
. Assumption :

+ Isotropic

+ Homogeneous

1 Prove

$$K = \frac{E}{3(1-2\nu)}$$



*. Only σ_1:

$$d\varepsilon_1 = \frac{d\sigma_1}{E} \;;\; d\varepsilon_2 = \frac{-\nu}{E} d\sigma_1 \;,\; d\varepsilon_3 = -\frac{\nu}{E} d\sigma_1$$

. Only σ_2:

$$d\varepsilon_1 = \frac{-\nu d\sigma_2}{E} \;;\; d\varepsilon_2 = \frac{d\sigma_2}{E} \;;\; d\varepsilon_3 = \frac{-\nu}{E} d\sigma_2$$

. Only σ_3:

$$d\varepsilon_1 = \frac{-\nu d\sigma_3}{E} \;;\; d\varepsilon_2 = \frac{-\nu d\sigma_3}{E} \;;\; d\varepsilon_3 = \frac{d\sigma_3}{E}$$

* Sum:

$$d\varepsilon_1 = \frac{1}{E} d\sigma_1 - \frac{\nu}{E} d\sigma_2 - \frac{\nu}{E} d\sigma_3$$

$$d\varepsilon_2 = -\frac{\nu}{E} d\sigma_1 + \frac{1}{E} d\sigma_2 - \frac{\nu}{E} d\sigma_3$$

$$d\varepsilon_3 = -\frac{\nu}{E} d\sigma_1 - \frac{\nu}{E} d\sigma_2 + \frac{1}{E} d\sigma_3$$

* Isotropic compression: $d\sigma_1 = d\sigma_2 = d\sigma_3$

$$\Rightarrow d\varepsilon_1 = d\varepsilon_2 = d\varepsilon_3 = \frac{1-2\nu}{E} d\sigma$$

$$\Rightarrow d\sigma_{1,2,3} = \frac{E}{1-2\nu} d\varepsilon_{1,2,3}$$

. $K = f(E, \nu)$

$$\Rightarrow dp = \boxed{\frac{E}{3(1-2\nu)}} d\varepsilon_v \qquad \begin{cases} dp = \frac{1}{3}(d\sigma_1 + d\sigma_2 + d\sigma_3) \\ d\varepsilon_v = d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 \end{cases}$$

. $\nu = 0.5 \Rightarrow$ incompressible material

K (bulk modulus)

2. Derive 3D elastic stiffness matrix:

. Derive the relation between stress σ and strain ε via ν, E, G

- Shear strain:

$$\gamma_{xy} = \gamma_{yx} = \frac{\tau_{xy}}{G} = \frac{\tau_{yx}}{G}$$

$$\gamma_{yz} = \gamma_{zy} = \frac{\tau_{yz}}{G} = \frac{\tau_{zy}}{G}$$

$$\gamma_{xz} = \gamma_{zx} = \frac{\tau_{xz}}{G} = \frac{\tau_{zx}}{G}$$

- Also, from (1), we have the equations of $d\varepsilon_1 . d\varepsilon_2 . d\varepsilon_3$

$$\begin{bmatrix} \dot{\varepsilon}_{xx} \\ \dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{zz} \\ \dot{\gamma}_{xy} \\ \dot{\gamma}_{yz} \\ \dot{\gamma}_{zx} \end{bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix} \begin{bmatrix} \dot{\sigma}_{xx} \\ \dot{\sigma}_{yy} \\ \dot{\sigma}_{zz} \\ \dot{\tau}_{xy} \\ \dot{\tau}_{yz} \\ \dot{\tau}_{zx} \end{bmatrix}$$

. Elastic stiffness matrix

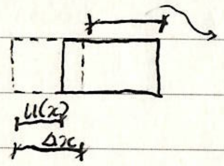
- Stiffness matrix: $\dot{\sigma} = D^{e} \dot{\varepsilon}$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} E'(1-\nu) & E'\nu & E'\nu & 0 & 0 & 0 \\ E'\nu & E'(1-\nu) & E'\nu & 0 & 0 & 0 \\ E'\nu & E'\nu & E'(1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{bmatrix}$$

Elastic Stiffness Matrix

Prove: $\varepsilon_x = \frac{\partial u}{\partial x}$; $\varepsilon_y = \frac{\partial v}{\partial y}$; $\varepsilon_z = \frac{\partial w}{\partial z}$

Strain $\varepsilon(x) \in$ position
Displacement $u(x) \in$ position
Question: How do we get strain field from displacement



$$u(x+\Delta x) = u(x) + \Delta x \frac{\partial u}{\partial x} + O(\Delta x^2)$$

$$\varepsilon_x = \frac{\Delta l}{l} = \frac{1}{\Delta x}(L' - L)$$

- $L' = \Delta x + u(x+\Delta x) - u(x)$

$$= \Delta x + \left[u(x) + \Delta x \frac{\partial u}{\partial x} + O(\Delta x^2) \right] - u(x)$$

$$= \Delta x + \Delta x \frac{\partial u}{\partial x} + O(\Delta x^2)$$

- $L = \Delta x$

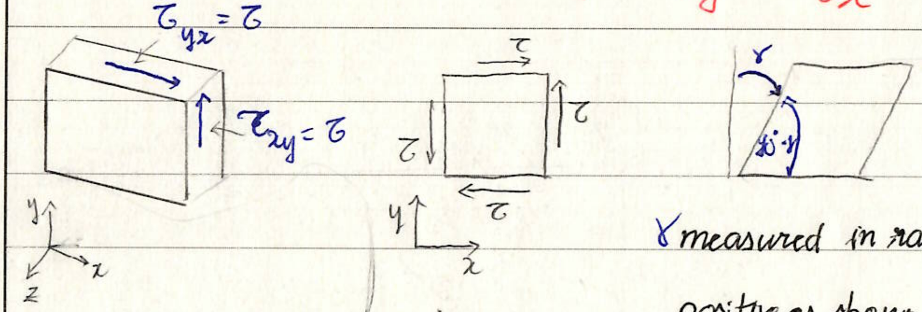
$$\Rightarrow \varepsilon_x = \frac{L - L'}{\Delta x} = \frac{\partial u}{\partial x} + O(\Delta x) \simeq \frac{\partial u}{\partial x}$$

Similarly: $\varepsilon_y = \frac{\partial v}{\partial y}$; $\varepsilon_z = \frac{\partial w}{\partial z}$

Normal strain measures changes in length along a specific direction

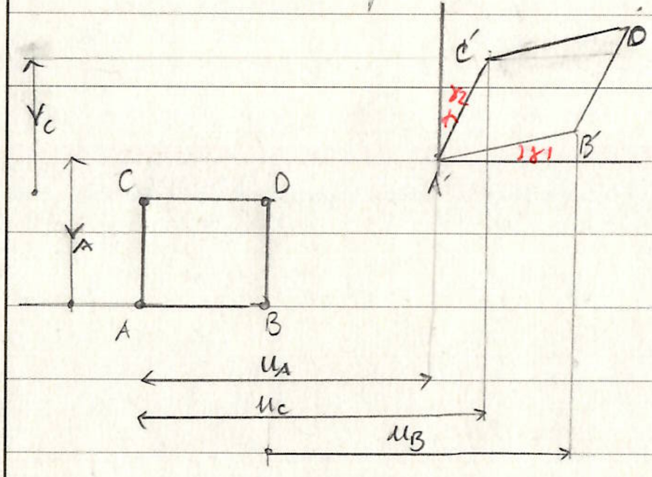
"Shear strain measures changes in angles with respect to two specific directions as the material distorts in response to shear stress"

Prove : Shear strain: $\gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}$



γ measured in radians.

positive as shown



Shear deformed:

$\gamma = \gamma_1 + \gamma_2$

$$\tan\gamma_1 = \frac{v_{B'} - v_{A'}}{u_{B'} - u_{A'}} = \frac{\Delta v_{BA}}{\Delta x + \Delta u_{BA}} \; ; \; \tan\gamma_2 = \frac{u_{C'} - u_{A'}}{v_{C'} - v_{A'}} = \frac{\Delta u_{CA}}{\Delta y + \Delta v_{CA}}$$

. Assume infinisimal strain

$$\Rightarrow \begin{cases} \gamma_1 \ll 1 \; ; \; \gamma_2 \ll 1 : \tan\gamma_1 \simeq \gamma_1 \; ; \; \tan\gamma_2 \simeq \gamma_2 \\ \Delta u_{BA} \ll \Delta x \Rightarrow \Delta x + \Delta u_{BA} \simeq \Delta x \\ \Delta v_{CA} \ll \Delta y \Rightarrow \Delta y + \Delta v_{CA} \simeq \Delta y \end{cases}$$

$$\gamma_{xy.av} = \gamma = \gamma_1 + \gamma_2 = \frac{\Delta v_{BA}}{\Delta x} + \frac{\Delta u_{CA}}{\Delta y} = \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta y}$$

. To define the shear strain γ_{xy} at one point P we pass the limit in the average strain expression by shrinking both dimensions Δx and Δy to zero:

$$\Rightarrow \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$