FLOW RULE

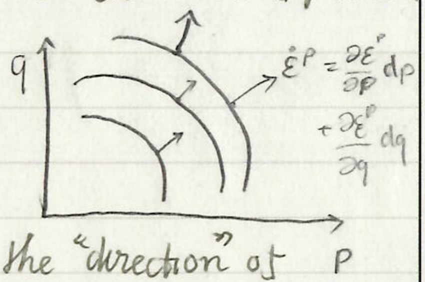
1 Plastic flow direction

- In 1D problem, it is clear that plastic strains take place in the direction of the applied stress
- In 2D, 3D we need to make a hypothesis regarding the direction of plastic flow

2 Plastic flow rule

- When the state of stress reaches yield criteria, the material undergoes plastic deformation. This is called plastic flow. In the theory of plasticity, the direction of plastic strain vector is defined through a flow rule by assuming the existence of a plastic potential function "g" to which the incremental strain vector are orthogonal. !
- The increments of plastic strains can be expressed by:

$$\dot{\varepsilon}^p_{ij} = \underbrace{\dot{\lambda}}_{\text{affect the magnitude of } \dot{\varepsilon}^p_{ij}} \underbrace{\frac{\partial g}{\partial \sigma_{ij}}}_{\text{control the "direction" of plastic deformation } (\perp g)}$$

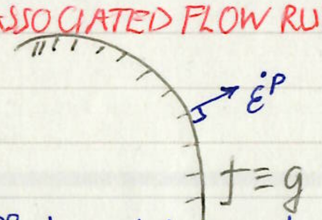


$$\dot{\varepsilon}^p_{ij} = \underbrace{\dot{\Lambda}}_{\text{magnitude}} \underbrace{\frac{\frac{\partial g}{\partial \sigma_{ij}}}{\left\| \frac{\partial g}{\partial \sigma_{ij}} \right\|}}_{\text{unit direction, no magnitude}}$$

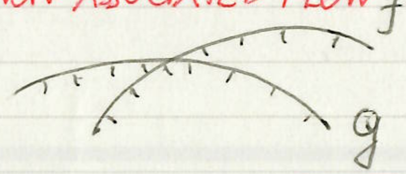
3 Associated & non-associated flow rule:

- Yield function (f) and plastic potential (g) are generally different functions
- If $f \equiv g \Rightarrow$ Associated flow rule

ASSOCIATED FLOW RULE



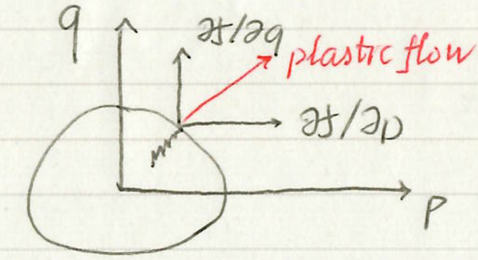
NON-ASSOCIATED FLOW RULE



① Plastic deformations depend on the $\boxed{\text{stress state}}$, not the $\boxed{\text{increment of the stress applied}}$

② Flow rule defines direction of plastic strain increment.

FLOW RULE



① Why Λ can be used the same for every component in stress & strain?

→ isotropic hardening is assumed

② $\dot{\varepsilon}^p_d$

q has a strong relation with shape deform

→ it should have a relationship with deviatoric strain

$$\dot{\varepsilon}^p_d = \dot{\Lambda} \frac{\partial g}{\partial q}$$

③ $\dot{\varepsilon}^p_v$

p has a strong relation with volumetric deform

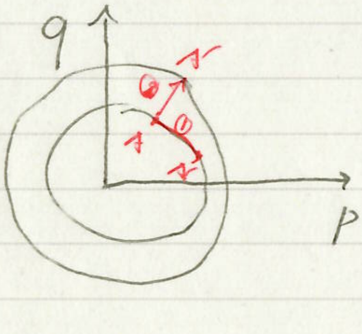
$$\dot{\varepsilon}^p_v = \dot{\Lambda} \frac{\partial g}{\partial p}$$

④ How do we define the magnitude of $\dot{\Lambda}$

No extra assumption is needed.

When we have plastic strain: $df = 0$ (*)

consistency condition



① Point A moves along yield surface

② Yield function expand from (1) & (2)

"Plastic strain has a strong relation with hardening behavior"

$$df = 0 \Leftrightarrow \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial H} \frac{\partial H}{\partial \varepsilon^p_{ij}} d\varepsilon^p_{ij} = 0 \qquad d\varepsilon^p_{ij} \rightarrow \Lambda \frac{\partial f}{\partial \sigma_{ij}}$$

$$\Rightarrow \Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij}}{\underbrace{\frac{\partial f}{\partial H} \frac{\partial H}{\partial \varepsilon^p_{ij}} \frac{\partial f}{\partial \sigma_{ij}}}_{\text{scalar}}}$$

Note: When there is no hardening behavior: $\frac{\partial f}{\partial H} \frac{\partial H}{\partial \varepsilon^p_{ij}} \frac{\partial f}{\partial \sigma_{ij}} \rightarrow 0$

we can't use eq. (*)