ELASTO - PLASTIC MODEL

① Assumption:

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p \qquad (1)$$

in fact, we cannot seperate elastic & plastic strain

② $\sigma - \varepsilon^e$

Effective stress has a direct relationship with ε^e

$$d\sigma = D^e \boxed{d\varepsilon^e} ? \qquad (2)$$

Note:

$$d\varepsilon = \begin{array}{l} \\ \\ \\ \text{Engineering } \gamma_{12} = \\ \text{strain } \gamma_{13} = \\ \gamma_{23} = \end{array} \begin{bmatrix} d\varepsilon_{11} \\ d\varepsilon_{22} \\ d\varepsilon_{33} \\ 2\,d\varepsilon_{12} \\ 2\,d\varepsilon_{13} \\ 2\,d\varepsilon_{23} \end{bmatrix} ; \quad d\sigma = \begin{bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ d\sigma_{12} \\ d\sigma_{13} \\ d\sigma_{23} \end{bmatrix} ; \quad D^e_{ijkl} = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

(ν, E, G, K)

Prove that: $\varepsilon = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{bmatrix}$ $\quad \frac{\partial f}{\partial \sigma} = \begin{bmatrix} \partial f/\partial\sigma_{11} \\ \partial f/\partial\sigma_{22} \\ \partial f/\partial\sigma_{33} \\ 2\partial f/\partial\sigma_{23} \\ 2\partial f/\partial\sigma_{31} \\ 2\partial f/\partial\sigma_{12} \end{bmatrix}$

③ Yield function:

$$f = f(\sigma_{ij}, H) \qquad (3)$$

H: hardening function /parameter

+ not related with stress
+ variables that may effect ε^p
+ such as: temperature, unsaturated..

④ Flow rule:

$$d\varepsilon^p_{ij} = \underbrace{\Lambda}_{\text{scalar, } \geq 0} \overbrace{\boxed{\frac{\partial g}{\partial \sigma_{ij}}}}^{\text{effect magnitude}} \; ; \; g: \text{potential energy function}$$

← control direction, with magnitude

Associated flow rule: $g = f \Rightarrow d\varepsilon^p_{ij} = \Lambda \frac{\partial f}{\partial \sigma_{ij}} \qquad (4)$

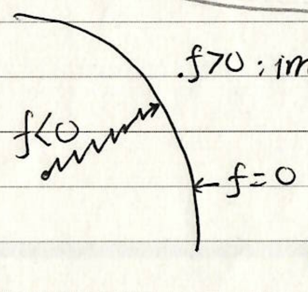
(1) When the state of stress reach yield criteria, the material undergoes plastic deformation. This is called plastic flow. In the theory of plasticity, the direction of plastic strain vector is defined through a flow rule by assuming the existence of a plastic potential function, to which the incremental strain vector are orthogonal. Then the increments of plastic strain can be expressed by $\Lambda \cdot \frac{\partial g}{\partial \sigma_{ij}}$

For some material, the plastic potential g and the yield function can be assumed to be the same → associated flow rule.

$$\frac{\partial f}{\partial \sigma_{ij}} = \begin{bmatrix} \partial f/\partial\sigma_{11} \\ \partial f/\partial\sigma_{22} \\ \partial f/\partial\sigma_{33} \\ 2\partial f/\partial\sigma_{23} \\ 2\partial f/\partial\sigma_{31} \\ \partial f/\partial\sigma_{12} \end{bmatrix}$$

⑤ Consistency condition:

When ε^p occurs, the soil's state needs to stay on the yield surface $\Leftrightarrow df = 0$



f>0: impossible $\quad \Leftrightarrow \boxed{\frac{\partial f}{\partial \sigma_{ij}}} d\sigma_{ij} + \frac{\partial f}{\partial H} \frac{\partial H}{\partial \varepsilon^p_{ii}} d\varepsilon^p_{ii} = 0 \qquad (5)$

{. Partial differentiation
. How much are the increments in $d\sigma_{ij}$, dH, $d\varepsilon^p_{ii}$ affect on the increment in f}

$$(1)(2)(4) \Rightarrow d\varepsilon_{ij} = D^{e-1}_{ijkl}\, d\sigma_{kl} + \Lambda \frac{\partial f}{\partial \sigma_{kl}}$$

ELASTO-PLASTIC MODEL

$$(4)(5) \Rightarrow \frac{\partial f}{\partial \sigma_{ij}}^T d\sigma_{ij} + \frac{\partial f}{\partial H}\frac{\partial H}{\partial \varepsilon^p_{ij\,ii}} \Lambda \frac{\partial f}{\partial \sigma_{ii}} = 0$$

$$\Rightarrow \quad \Lambda = \frac{(\partial f/\partial\sigma_{ij})^T \cdot d\sigma_{ij}}{-\frac{\partial f}{\partial H}\frac{\partial H}{\partial \varepsilon^p_{ij}}\frac{\partial f}{\partial \sigma_{ii}}}$$

* ~~when ε^p occurs~~ Notice that $\Rightarrow \frac{\partial f}{\partial \sigma_{ij}} = 0$ (critical state) so it is not stable to find Λ this way

$$* \left\{\frac{\partial f}{\partial \sigma_{ij}}\right\}^T D^e_{ijkl}\, d\varepsilon_{kl} = \left\{\frac{\partial f}{\partial \sigma_{ij}}\right\}^T D^e_{ijkl}\left(d\varepsilon^e_{kl} + d\varepsilon^p_{kl}\right)$$

$$= \left\{\frac{\partial f}{\partial \sigma_{ij}}\right\}^T d\sigma_{ij} + \left\{\frac{\partial f}{\partial \sigma_{ij}}\right\}^T D^e_{ijkl}\frac{\partial f}{\partial \sigma_{kl}}\Lambda$$

$$= -\frac{\partial f}{\partial H}\frac{\partial H}{\partial \varepsilon^p_{ij\,ii}}\Lambda\frac{\partial f}{\partial \sigma_{ii}} + \left\{\frac{\partial f}{\partial \sigma_{ij}}\right\}^T D^e_{ijkl}\frac{\partial f}{\partial \sigma_{kl}}\Lambda$$

$$\Rightarrow \quad \Lambda = \frac{\left(\frac{\partial f}{\partial \sigma_{ij}}\right)^T D^e_{ijkl}\, d\varepsilon_{kl}}{-\frac{\partial f}{\partial H}\frac{\partial H}{\partial \varepsilon^p_{ij\,ii}}\frac{\partial f}{\partial \sigma_{ii}} + \left(\frac{\partial f}{\partial \sigma_{ij}}\right)^T D^e_{ijkl}\frac{\partial f}{\partial \sigma_{kl}}}$$

$$* \quad d\sigma_{ij} = D^e_{ijkl}\, d\varepsilon^e_{kl} = D^e_{ijkl}\left(d\varepsilon_{kl} - d\varepsilon^p_{ij}\right)$$

$$= D^e_{ijkl}\left(d\varepsilon_{kl} - \Lambda\frac{\partial f}{\partial \sigma_{ij}}\right)$$

$$= D^e_{ijkl}\left[d\varepsilon_{kl} - \frac{\partial f/\partial\sigma_{ij}\; D^e_{mnop}\left\{\partial f/\partial\sigma_{op}\right\}\left\{\partial f/\partial\sigma\right\}^T D^e\, d\varepsilon_{kl}}{\underbrace{-\frac{\partial f}{\partial H}\frac{\partial H}{\partial \varepsilon^p_{ij\,ii}}\frac{\partial f}{\partial \sigma_{ii}}}_{=A} + \left\{\frac{\partial f}{\partial \sigma_{ij}}\right\}^T D^e_{ijkl}\frac{\partial f}{\partial \sigma_{kl}}}\right]$$

$$d\sigma_{ij} = \left[D^e_{ijop} - \frac{D^e_{ijkl}\left\{\frac{\partial f}{\partial \sigma_{kl}}\right\}\left\{\frac{\partial f}{\partial \sigma_{mn}}\right\}^T D^e_{mnop}}{\left\{\frac{\partial f}{\partial \sigma_{ij}}\right\}^T D^e_{ijkl}\left\{\frac{\partial f}{\partial \sigma_{kl}}\right\} + A}\right] d\varepsilon_{op}$$